RESISTANCE COMPUTATION IN COMPOUND CHANNELS

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ABSTRACT

Resistance to flow past a cross-section is normally computed using a single representative parameter such as the Manning n, which can be estimated from extensive well established links to bed surface characteristics. However in many channels, the surface roughness may vary considerably within a single cross-section, making a single Manning n hard to assess. A popular method of establishing a suitable composite roughness parameter is to allow a “relative roughness” to be specified, so that many local estimates of the roughness parameter can be made by standard methods and then averaged to convey the cumulative effects of the section bed as a whole. This is shown to rely on the traditional hydraulic approach to analysis of multi-dimensional flows within channels. The averaging process can be formulated in many ways, with the resulting composite resistance varying drastically, so the reasons for using the usual Lotter method are discussed. However, it is shown that this method still has some unsatisfactory features, such as a possible decrease in composite resistance in response to an increase in relative roughness. A solution based on perimeter adjustment is then proposed and shown to avoid previous distortions.

KEYWORDS

***AULOS*, Resistance, roughness, compound channels, relative roughness, Manning n, Lotter method**

1 INTRODUCTION

Many hydraulic modelling applications encounter channels with considerable variation of bed roughness over a cross-section, so there has been considerable demand for the implementation of compound channel facilities in the *AULOS* package. On undertaking research into historical methods of analysis, many treatments were found to be inadequate in that they failed to base assumptions on sound physical principles. Worse, beta testing of the coded version of the widely used Lotter analysis showed up considerable problems which are contrary to commonsense - for example, a decrease in composite resistance sometimes resulted from increases in relative resistance coefficients.

These problems and their solution are of general interest, so are presented here.

2 HYDRAULICS VERSUS ONE-DIMENSIONAL ANALYSIS

The general solution of flow problems in four dimensions is not yet tractable analytically, so simplifying assumptions are needed. The earliest simplifying assumptions were those of hydraulics and of one-dimensional analysis. These are distinct approaches, although they are often confused, as in their simplest form they become identical.

2.1 HYDRAULIC ANALYSIS

Hydraulics (implied by the origins of the word in the Greek *hudor* water + *aulos* pipe) views all flow as passing through elements shaped as streamtubes, in which the lateral boundary walls are aligned with the instantaneous local flow direction, while the two end boundaries are perpendicular to the flow direction, and therefore to the boundary walls. Flow enters the element only through the upstream end boundary, and leaves only through the downstream end boundary – the lateral boundaries move with any lateral movement of the fluid. This hydraulic approach (cf. the old graphical “flow-net”analysis) is applicable to flow problems in the full four dimensions (three spatial plus time), but obviously provides effective simplifications mainly where boundary walls can be found which behave in some regular fashion. The hydraulic approach has been successful in particular where boundary walls remain nearly fixed in space and run approximately parallel to each other to form channels (understood here to include pipes, but also to extend to natural watercourses with pipe-like characteristics). In such cases, the end boundary surfaces can be treated as plane cross-sections, lying perpendicular to the mean flow direction through them.

No inconsistency in this analysis arises if the channels are not straight either in elevation or plan, or if additional equations continue to be used to analyse flow variations in the lateral and vertical dimensions over the cross-section.

The hydraulic approach is most powerful where bulk characteristics of the cross-section can be related, rather than local point values. A simple example is the discharge Q, which in steady conditions is constant throughout the length of the channel. A more subtle case is the Bernoulli head H. In contrast, the local point value of velocity V may vary drastically along a channel if the depth of flow is irregular.

2.2 ONE-DIMENSIONAL ANALYSIS

One-dimensional analysis places a straight line through the flow, and attempts to analyse conditions in terms of the local flow properties along this axis. (Conventionally, the axis line is purely spatial; the treatment of time as the "fourth dimension"is relatively recent, and is certainly not required for steady flow problems. Of course, the time dimension is fundamental in unsteady flow, but the extra dimension is taken as implicit in the use of the word "unsteady", so that a "one-dimensional unsteady" analysis is taken to mean analysis in one spatial dimension plus the time dimension).

Strictly, one dimensional analysis will be applicable only where local flow properties do not vary over a cross-section, now defined as any plane perpendicular to the axis, but this is conventionally relaxed to allow variation of the position of the flow boundaries over a vertical plane through that axis. The bottom of the flow is defined by the bed surface. The top of the flow may be a fixed surface, for pressure flow in a pipe, or a free surface in open channel flow. In the latter case, the depth of flow is seen as one of the local flow properties along the axis, instead of counting as a distance from the axis in a second (vertical) dimension. This partial extra dimension is an example of a "fractal"dimension.

Therefore the main test is the presence or absence of equations explicitly defining analysis in any perpendicular dimension – if these require to be solved to complete the flow description, then the analysis cannot be one-dimensional.

Summarising, then, hydraulic analysis deals with multi-dimensional flows through channels, while one-dimensional analysis deals with longitudinal variations of local flow properties along a single axis.

3 COMPOUND CHANNELS

Compound channels are a concept used to deal with significant variations in flow conditions across a cross-section. Because the variations are lateral, they are beyond the scope of one-dimensional analysis, but are still treatable by hydraulic analysis. These flow variations are typically caused by changes in the bed resistance, often associated with the growth of vegetation which is well adapted to channel berm conditions, but which cannot establish itself in the permanently submerged conditions of the low flow channel.

Examples are shown in Photographs 1 and 2.

3.1 THE RANGITATA DIVERSION RACE

Photograph 1 shows the Rangitata Diversion Race, an artificial channel in Mid-Canterbury which has been designed with protective stopbanks set back, so that a berm area will come into use under conditions of high flow or high backwater. These are relatively rare, as the pictured surge test was run near maximum flow conditions within the normal operational range, and the canal surge is just spilling onto the berms.

Photograph 1:   Surge Tests on the Rangitata Diversion Race

Photograph 2:   Tidal Reach of Cooks Stream

Should the canal need to be modelled under berm flow conditions, the same roughness value is likely to represent both berms, as the vegetation cover is visually identical. However the roughness for the central canal is unlikely to be similar, as the canal bed is quite unlike the berm. Any single roughness characterization is therefore difficult to assess, as the importance of the central canal bed resistance can be expected to diminish as the water level rises across the berms.

3.2 COOKS STREAM

Photograph 2 shows Cooks Stream in Cooks Beach. This is a natural channel, but here the berm areas are partly tidal, although high flood levels can be expected to exceed the tidal range. In this case the vegetation on the right and left sides of the low flow channel is quite dissimilar, so multiple roughness characterization would be needed to represent the total channel resistance to flow.

3.3 RELATIVE ROUGHNESS

Whatever method is used to represent bed roughness, it is quite easy to develop a relative roughness concept, which assumes a value of 2 on the berm means simply “the berm is twice as rough as the central low flow channel.” However, this has to be interpreted with care to become a quantitative relationship rather than some kind of value judgement.

The Manning equation is widely accepted for the evaluation of resistance, at least in fully turbulent flow conditions. This can be related (Henderson (1966)) to $D^{1/6}$, where D is a grain size parameter. The application of relative roughness to the Manning n parameter then means that “twice as rough” implies a grain size multiplied by 64! Fortunately, the wide availability of illustrated tables of Manning n (Ven Te Chow 1959, Hicks & Mason 1991) related to various types of vegetation cover means that it is practical to assess relative roughness directly in terms of Manning n ratios without direct consideration of grain size. This is particularly important where the resistance elements are dominated by vegetation or by bedforms, where “grain size” is not applicable.

4 BALANCING PRINCIPLES

Many ways can be found to assess the effect of roughness variations on the total flow through a cross-section. This is still an active area of research, with methods such as the k-ε model showing promise. However, such methods are still restricted in their application, and are therefore beyond normal hydraulic engineering practice.

Standard methods rely on some balancing principle to enable the whole section flow to be distributed correctly among subsections. A selection of these methods is discussed by Ven te Chow (1959), which should be consulted for the original references.

4.1 EQUAL VELOCITY METHODS

Horton and Einstein both assumed that each subsection has the same mean velocity. However, there seems no good physical principle why this should be so, and many cases are observed where this is clearly not so.

4.2 FORCE SUMMATION METHODS

Pavlovskii, Mühlhofer, and Einstein and Banks assumed that the total resistance force on a section was that found by summing the resistance forces in each subsection. While this is fundamentally true, application as proposed results in different slopes in each subchannel, which cannot be sustained without rebalancing by exchange of flows. These carry momentum, which alters the force balance for each subsection, invalidating the original assumption that the local resistance force is a function only of the local bed roughness.

4.3 EQUAL SLOPE METHODS

Lotter assumed that the total flow is equal to the sum of the discharges in each subsection. Again, this is fundamentally true, but as shown by Henderson (1966), the important feature of Lotter's method was the assumption of an equal slope for calculating the discharge in all subsections.

According to Ven Te Chow (1959), the Lotter formula is:

$$n = \frac{PR^{5/3}}{\sum_{1}^{N}\left(\frac{P_N R_N^{5/3}}{n_N}\right)} \qquad (1)$$

where R_1, R_2, …, R_N are hydraulic radii of the subdivided areas while the unsubscripted R is the hydraulic radius of the whole section. Similar conventions apply for the wetted perimeter P and Manning n. How to calculate the subsection hydraulic radii is not defined, except for simple channel sections, where it is advised that the relationship

$$R_1 = R_2 = ... = R_N = R$$

may be assumed.

Henderson (1966) specified that, for the same value of friction slope S_f to apply throughout

$$\frac{Q_i}{K_i} = S_f^{1/2}$$

where Q_i is the discharge through the ith subsection and K_i is the ith subsection "conveyance", defined using the Manning formula as

$$K_i = \frac{MA_i R_i^{2/3}}{n_i}$$

Here M is a constant related to the dimensional units used (= 1.49 in the foot-second units used by Henderson), while A_i, R_i and n_i are respectively the area, hydraulic radius and Manning n applying to the ith subsection. Further, from the context it is clear that $R_i = A_i/P_i$, where P_i is the wetted perimeter of the ith subsection.

It follows from the above that, using corresponding definitions for the full section Q and K,

$$\frac{Qn}{MAR^{2/3}} = S_f^{1/2} = \frac{Q_i n_i}{MA_i R_i^{2/3}}$$

Therefore, summing the discharges through all sections as per Lotter,

$$\sum_{i=1}^{N} Q_i = Q = \frac{Qn}{AR^{2/3}} \sum_{i=1}^{N} \frac{A_i R_i^{2/3}}{n_i}$$

giving

$$n = \frac{AR^{2/3}}{\sum_{i=1}^{N} \frac{A_i R_i^{2/3}}{n_i}} = \frac{PR^{5/3}}{\sum_{i=1}^{N} \frac{P_i R_i^{5/3}}{n_i}}$$

This recovers Lotter's formula, but with the subsection hydraulic radii now clearly defined in terms of the area and wetted perimeter. Although not intrinsic to the derivation, the definition of the subsection boundaries is also a requirement for practical computation of A_i. In practice, these boundaries are taken as vertical lines above the point at which the Manning n changes on the bed, as any other definition is difficult to express for a general irregular cross-section.

Where the subsection roughnesses are expressed relative to some standard Manning n value n_S, it is convenient to introduce the parameters r, r_i for the relative roughness, so that

$$n = r n_S , \quad n_i = r_i n_S \tag{2}$$

Here r refers to the relative resistance of the full section, as computed from Equation (1).

Using (2), we can eliminate n and n_i from the Henderson expression to give

$$r = \frac{AR^{2/3}}{\sum_{i=1}^{N} \frac{A_i R_i^{2/3}}{r_i}} \tag{3}$$

Using this form of the equation, some potential problems with the analysis can be shown to emerge.

5 PROBLEMS WITH THE HENDERSON-LOTTER ANALYSIS

5.1 EFFECT OF SECTION SUBDIVISION

Take as an example a rectangular cross-section, 3m wide and 1m deep. For this, A=3m^2 and P=5m, so R = 0.6m and $AR^{2/3}$ = 2.134m$^{8/3}$. If the channel is divided into two equal subsections, A=1.5m^2 and P=2.5m for each half. Therefore R = 0.6m again for both halves and $\Sigma A_i R_i^{2/3}$ = 2.134m$^{8/3}$ – the same value as for the full channel.

However, if the channel is divided into three equal subsections (that is, equal with respect to width), A=1m^2 for each third, while P=2m for the outside thirds and 1m for the middle third, leading to $\Sigma A_i R_i^{2/3}$ = 2.260m$^{8/3}$.

This gives the unwelcome result that r = 0.944 for all r_i = 1, where a user of the computation would intuitively expect r = 1. In other words, subdivision alone can change the value of the cross-section resistance even when all relative resistance values are left at the default value of 1!

5.2 PROBLEMS WITH UNIFORM CHANGES IN RELATIVE RESISTANCE

Suppose the relative resistance is uniformly set r_i = 2 for all subsections. Then for the above example with the channel divided into thirds, the value of $\Sigma A_i R_i^{2/3}/r_i$ is 1.130m$^{8/3}$. This gives a value of r of 1.888, where this would be expected to be 2.

However, consideration of the form of equation (3) shows that the problem is again with the section subdivision, rather than with the value of relative resistance, as this value of r is exactly double 0.944, that obtained for the relative resistance value of 1.

Both problems can therefore be removed by accumulating subsections with the same relative roughness and treating them as a single section. Formally, this means that if M adjacent subsections are found to have the same relative resistance, they are accumulated into the ith subsection using the formula

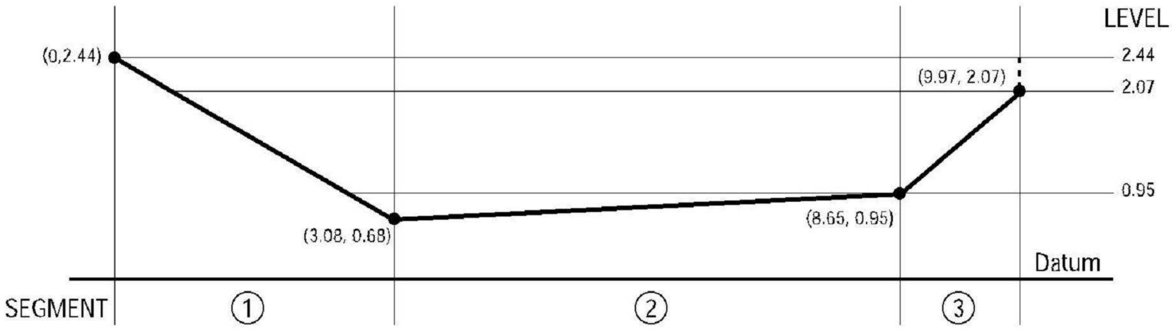
$$A_i R_i^{2/3} = \frac{\left(\sum_{j=1}^{M} A_j\right)^{5/3}}{\left(\sum_{j=1}^{M} P_j\right)^{2/3}} \qquad (4)$$

It can easily be seen that if this formulation is used in setting up the compound channel algorithm, equation (3) will always give $r = r_i$ for constant values of r_i over the whole section as desired.

5.3 INCREASED RELATIVE RESISTANCE CAN MEAN REDUCED RESISTANCE!

As found in a subsequent model study used for beta testing, this reformulation does not guarantee satisfactory performance where r_i varies across the section. The section data plotted in Figure 1 was surveyed in Cooks Creek, near the scene shown in Photograph 2.

Figure 1: Problem Cross-Section (found in an actual model study)



An attempt was made to represent greatly increased vegetative roughness in the subsection labelled Segment 3 by setting the relative roughness to 3.0, with the standard value of 1 in Segments 1 and 2. This had the unexpected result of giving r = 0.995 for the level 2.07m from Equations (3), even after adjustment using the formulation in Equation (4).

For those wishing to work through the details, the 1st subsection is formed from Segments 1 and 2 and the second subsection from Segment 3. The summation results are

$$\frac{\sum_{j=1}^{3} A_j^{5/3}}{\sum_{j=1}^{3} P_j^{2/3}} = 8.9869978, \quad \frac{\sum_{j=1}^{2} A_j^{5/3}}{\sum_{j=1}^{2} P_j^{2/3}} = 8.8888335, \quad \frac{\sum_{j=3}^{3} A_j^{5/3}}{3\sum_{j=3}^{3} P_j^{2/3}} = 0.1397213$$

Then r = 8.9869978/9.0285548 to give the above result. This is a clearly unsatisfactory response to an increase in the relative resistance of segment 3 from 1 to 3.0, even though that segment is quite small!

6 MODIFIED HENDERSON-LOTTER ANALYSIS

The problem is still related to the effect of subdividing the cross-section, as illustrated above in Section 5.1. Where the areas and the wetted perimeters are divided in the same ratio, there is no problem, but where this does not happen, the summation process causes distortions, apparently mainly in the direction of reduced resistance.

One approach is that suggested by Ven Te Chow (Section 4.3), which is to take all the subsection hydraulic radius values as the same as the composite value. This effectively means the areas and wetted perimeters always have a fixed ratio to each other, but this seems unrealistically distorted where some subsections are berm flows of possible minimal depth while others are main channel flows of considerable depth.

It is equally defensible to distribute the area according to the wetted perimeter proportions, or the wetted perimeter according to the area proportions. Normally the wetted perimeter proportion will exceed the area proportion near the sides of a channel if there is a significant vertical contribution to the wetted perimeter (as in the example in 5.1 above), while the reverse will hold in central deep water sections.

Either method has the potential to cause considerable confusion as to exactly which subareas and subperimeters correspond with given subsections, so an alternative is proposed based on conveyance proportions, excluding the resistance term. This means (3) is replaced by

$$r = \frac{\sum_{i=1}^{N} A_i R_i^{2/3}}{\sum_{i=1}^{N} \frac{A_i R_i^{2/3}}{r_i}} \qquad (5)$$

This formulation, still used in conjunction with (4), can be seen to guarantee that any increase in any of the r_i will result in an increase in r as is intuitively to be expected.

A practical issue is the problem that there will be a different value of *r* computed for every level (see Figure 1), and this requires an array to be set up at run time. Also there is the question of how *r* is to be interpolated between levels for evaluation of the friction slope at an arbitrary water level.

A tidy solution computationally is to modify the values of *A* or *P* so as to have the same effect as a change in *r* when the friction slope is calculated. Modification of *A* is known to have been tried, but this is not considered good practice because of the risk of corrupting other computations relying on *A*, such as the discharge/velocity relationship or the calculation of wave celerity. Also *A* varies quadratically between levels, whereas a simpler linear interpolation is all that can be reasonably justified for *r*.

Modification of *P* carries none of these disadvantages: it is used only in the calculation of friction slope, and it varies linearly between levels, at least in a segmented cross-section. Accordingly, the values of *r* are conveyed by modified wetted perimeters in *AULOS*.

The required modification can be determined by consideration of the expression for friction slope:

$$S_f^{1/2} = \frac{nQ}{MAR^{2/3}} = \frac{rn_S QP^{2/3}}{MA^{5/3}} = \frac{n_S QP'^{2/3}}{MA'^{5/3}}$$

Here *P'* and *A'* are *P* and *A* modified to account for the effect of *r*. Given that modifications of *A* have been rejected, *A'* = *A*, so finally

$$P'^{2/3} = rP^{2/3}, \qquad P' = r^{3/2}P \qquad (6)$$

A table of *P'* vs level can therefore be set up at run time, and interpolated linearly as usual for *P*. This also offers the pleasing physical interpretation of the wetted perimeter as corrugated, with the height of the corrugations increasing to provide the increase in *P'* corresponding with an increase in roughness.

Finally, it should be noted that this method is restricted to cross-sections which are open to above, at least in any wetted parts. This is because any overhangs cannot be treated by the adopted method of defining subsection boundaries by extending verticals upwards from the change of resistance on the bed. Ways could be found to deal with this, but it is extremely doubtful that the Lotter-Henderson approach is sufficiently accurate in such cases to justify the effort. Accordingly, sections with any such features are rejected.

7 CONCLUSIONS

Compound channels can be treated by assessing the lateral variation of resistance in cross-sections using traditional hydraulic techniques for analysis of multi-dimensional flows within channels.

In evaluating composite resistance, use of equations (4), (5) and (6) together have been shown to ensure that

1. Section subdivision alone has no effect.

2. Uniform changes in relative resistance have the same effect as multiplying the Manning n by the relative resistance in a simple single thread channel.

3. Increasing any relative resistance always increases the overall resistance.

4. Changes in resistance can be conveniently carried as modifications to the wetted perimeter.

The use of other alternatives can be prone to serious errors, even when based on the same Lotter analysis.

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